

# One-dimensional dynamics of $\text{QCD}_2$ string

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## Abstract

We show that  $\text{QCD}_2$  on 2D pseudo-manifolds is consistent with the Gross-Taylor string picture. It allows us to introduce a model describing the one-dimensional evolution of the  $\text{QCD}_2$  string (in the sense that  $\text{QCD}_2$  itself is regarded as a zero-dimensional system). The model is shown to possess the third order phase transition associated with the  $c = 1$  Bose string below which it becomes equivalent to the vortex-free sector of the 1-dimensional matrix model. We argue that it could serve as a toy model for the glueball-threshold behavior of multicolor QCD.

# 1 Introduction

QCD on 2-dimensional compact manifolds attracted attention for the first time in Ref. [1] and then was investigated from different viewpoints in Refs. [2-9] (the earlier papers not touching the topological aspects are [10, 11, 12]). The recent renewal of interest in 2-dimensional gauge theories was in a big part triggered by the Gross and Taylor stringy picture of  $\text{QCD}_2$  [3]. It appears that, in the large  $N$  limit, the spherical topology is distinguished from all the others [4]. In this case, continuous  $\text{QCD}_2$  undergoes the third order phase transition below which the model apparently admits of no stringy interpretation [5]. This transition also takes place on all simply-connected closed 2-dimensional pseudo-manifolds, the so-called homotopic bouquets of spheres. The simplest example of such a space is given by  $p$  disks whose boundaries are identified. The Euler character of this object equals  $p$ . Two disks give a sphere. If  $p \geq 3$ , we obtain the simply-connected pseudo-manifold  $\mathcal{P}_p$ .

The sum-over-coverings picture has limited validity for 2D spaces having a non-trivial second homotopy group,  $\pi_2$ . Let us consider the Wilson average for a simple closed loop:  $W(L)$ . If the loop shrinks,  $L \rightarrow \bullet$ , then  $W(\bullet) = 1$  in any gauge theory, while within a string model one finds a closed-string partition function with one puncture. As  $\pi_2 \neq 0$ , there is no geometrical reason for the string partition function to vanish [7]. Since all plaquette-made lattices can be regarded as 2D spaces with non-trivial  $\pi_2$ , this situation is not exotic.

As was discovered and Douglas and Kazakov [5], the nice stringy picture is spoiled when the manifold is a 2-sphere of a small enough area. Although it allowed for some speculations, the singleness of this example restricts very much our intuition with respect to possible guesses about more realistic physical systems. Fortunately, the consideration can be extended in two directions without losing exact solubility. First, one can consider  $\text{QCD}_2$  on the pseudo-manifolds  $\mathcal{P}_p$ . And second, one can make the model dynamical by introducing a continuous time direction along which the  $\text{QCD}_2$  string can propagate. Let us imagine a string theory in which no internal degrees of freedom can be exited. The only allowed processes are creation, destruction, splitting and joining of closed strings. As will be discussed later, such a toy model can be quite instructive and even help us to learn something about more realistic physical systems.

## 2 $\text{QCD}_2$ on pseudo-manifolds

According to the general rules [13, 10], we can construct the  $\text{QCD}_2$  partition function on  $\mathcal{P}_p$  by putting into correspondence to a disk of a (dimensionless) area  $A$  the  $U(N)$

heat-kernel

$$G_A(\Omega) = \sum_R d_R e^{-\frac{A}{2N} C_R} \chi_R(\Omega) \quad (1)$$

where  $\Omega \in U(N)$  is a holonomy along the disk boundary. We obtain the partition function on  $\mathcal{P}_p$  by identifying the holonomies and integrating over them:

$$Z_{\mathcal{P}_p}(A_1, \dots, A_p) = \int d\Omega \prod_{k=1}^p G_{A_k}(\Omega) = \sum_{R_1 \dots R_p} \prod_{k=1}^p \left( d_{R_k} e^{-\frac{A_k}{2N} C_{R_k}} \right) \int d\Omega \prod_{k=1}^p \chi_{R_k}(\Omega) \quad (2)$$

In Eqs. (1) and (2),  $R_1, \dots, R_p$  are  $U(N)$  irreps parametrized by lengths of rows in Young tables:

$$R \equiv [m_1, m_2, \dots, m_N] \quad m_1 \geq m_2 \geq \dots \geq m_N \quad (3)$$

Technically, it is more convenient to introduce the strictly ordered numbers

$$\ell_k = m_k + \frac{N+1-2k}{2}, \quad \ell_1 > \ell_2 > \dots > \ell_N \quad (4)$$

in terms of which the dimension of an irrep  $R$  takes the form

$$d_R = \prod_{i < j} \left( \frac{\ell_i - \ell_j}{j - i} \right) = \frac{\Delta(\ell)}{\Delta_0} \quad (5)$$

$\Delta(\ell)$  is the Van-der-Monde determinant;  $\Delta_0 = \Delta(\ell)|_{\ell_k=k}$ . The second Casimir eigenvalue is

$$C_R = \sum_{k=1}^N \ell_k^2 - \frac{N(N^2-1)}{12} \quad (6)$$

The first Weyl formula represents the character as the ratio of the two determinants:

$$\chi_R(e^{i\varphi}) = \frac{\det(e^{i\varphi_j \ell_k})}{\det(e^{i\varphi_j (\frac{N+1}{2} - k)})} \quad (7)$$

The integral of  $p$  characters gives the multiplicity of the trivial representation in the tensor product  $R_1 \otimes R_2 \otimes \dots \otimes R_p$ . This non-trivial factor is characteristic to the pseudo-manifolds in question. We want to show that its presence does not destroy the Gross-Taylor sum-over-coverings picture of QCD<sub>2</sub>. More precisely, we are going to show that, once one interprets the number of boxes in a Young table as the number of sheets of a

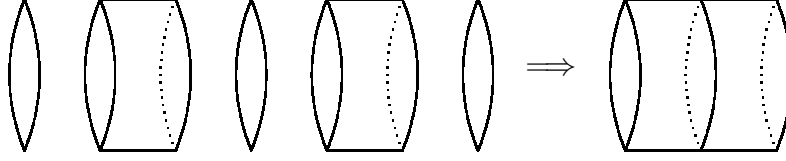


Figure 1: An example of a pseudo-manifold:  $\widehat{\mathcal{P}}_3$ .

covering, the multiplicities allow only for the tables compatible with the geometrical interpretation. It is important that the coverings are oriented. The change of an orientation of a disk corresponds to the conjugation of a representation attached to it:

$$(R = [m_1, m_2, \dots, m_{N-1}, m_N]) \Rightarrow (\tilde{R} = [-m_N, -m_{N-1}, \dots, -m_2, -m_1]) \quad (8)$$

The associativity of the tensor product means that it is sufficient to consider only the three irrep multiplicities

$$M_{R_1 R_2}^{R_3} = \int_0^{2\pi} \prod_{n=1}^N d\varphi_n |\Delta(e^{i\varphi})|^2 \chi_{R_1}(e^{i\varphi}) \chi_{R_2}(e^{i\varphi}) \overline{\chi_{R_3}(e^{i\varphi})} \quad (9)$$

Geometricly, it means that we can always deform a pseudo-manifold in such a way that only 3 disks meet on every boundary circle. One can imagine the corresponding construction as a number of cylinders glued pairwise along boundaries of disks (see an example in Figure 1). If the cylinders shrink to circles, the described above construction recovers.

The QCD<sub>2</sub> partition function on a cylinder of an area  $\varepsilon$  with fixed holonomies along its boundary loops is

$$Z_\varepsilon^{(1)}(\Omega_1, \Omega_2) = \sum_R e^{-\frac{\varepsilon}{2N} C_R} \chi_R(\Omega_1) \overline{\chi_R(\Omega_2)} \quad (10)$$

Thus we find for the deformed pseudo-manifold  $\widehat{\mathcal{P}}_p$

$$Z_{\widehat{\mathcal{P}}_p} = \sum_{R_1 \dots R_p} \sum_{S_1 \dots S_{p-1}} \prod_{k=1}^p \left( d_{R_k} e^{-\frac{A_k}{2N} C_{R_k}} e^{-\frac{\varepsilon}{2N} C_{S_k}} \right) \prod_{k=1}^p M_{R_k S_{k-1}}^{S_k} \quad (11)$$

where it is assumed that  $S_0 \equiv 0$  and  $S_p \equiv 0$  (hence  $S_1 = R_1$  and  $S_{p-1} = \widetilde{R}_p$ ).

We would like to treat coverings having opposite orientations separately. It is self-consistent only in the  $N \rightarrow \infty$  limit and only if one restricts the consideration to irreps for which the second Casimir is of the maximum order  $N$  [3]. Then the theory possesses 2 chiral sectors:

1) Irreps having a finite number of positive highest weight components

$$\{R_+\} : (m_1 \geq m_2 \geq \dots \geq m_n \geq m_{n+1} = m_{n+2} = \dots = m_N) \quad (12)$$

We shall associate irreps of this form with coverings of the positive orientation.

2) Irreps having a finite number of negative highest weight components

$$\{R_-\} : (0 = m_1 = \dots = m_{n-1} \geq m_n \geq \dots \geq m_N) \quad (13)$$

are associated with the inverse orientation. The trivial representation,  $\{0\}$ , corresponds to the empty configuration.

The general case is described by a product of 2 irreps from the different sectors:

$$R = r_+ \otimes r_-, \quad r_+ \in \{R_+\} \quad r_- \in \{R_-\} \quad (14)$$

A decomposition of  $R$  into irreducible representations is determined by  $r_+$  and  $r_-$ , and in turn, uniquely determines them provided the mentioned above conditions are fulfilled.

Associativity of the tensor product again allows us to restrict the consideration to irreps from only one of the sectors, because the change of an orientation permutes  $\{R_+\}$  and  $\{R_-\}$ . Let us consider the multiplicity  $M_{RS}^T$ , where  $R, S, T \in \{R_+\}$ . The corresponding pseudo-manifold consists of 3 cylinders glued along a circle. We fix 3 holonomies along the 3 components of the boundary. To remove the branching points, we put into correspondence to every cylinder the weight (cf. Eq. (10))

$$H(u, w) = \sum_{r \in \{R_+\}} \chi_r(u) \overline{\chi_r(w)} = \prod_{i,j=1}^N \frac{1}{1 - u_i \overline{w_j}} = e^{F(u, w)} \quad (15)$$

where

$$F(u, w) = \sum_{p=1}^{\infty} \frac{1}{p} \text{tr}(u^p) \overline{\text{tr}(w^p)} \quad (16)$$

is the generating function for  $p$ -fold unbranched connected coverings of a cylinder. Being exponentiated, it produces all possible coverings with equal weights and correct symmetry factors. It should be noted that these simple expression and clear interpretation exist only for coverings of a fixed orientation.

We have

$$H(u_1, w) H(u_2, w) = \sum_{R, S, T \in \{R_+\}} M_{RS}^T \chi_R(u_1) \chi_S(u_2) \overline{\chi_T(w)} \quad (17)$$

As  $\log [H(u_1, w) H(u_2, w)] = F(u_1, w) + F(u_2, w)$ , the appearing configurations are exactly all possible oriented coverings of the 3 glued cylinders. Thus we arrive at the desired interpretation of the multiplicities.

### 3 Continuum limit in the infinite chain

Let us consider the infinite pseudo-manifold of the type shown in Fig. 1, *i.e.*, the infinite in both directions chain of cylinders and disks. According to the results of the previous section, the QCD<sub>2</sub> partition function for this object allows for the string interpretation. One can regard this model as the ordinary lattice QCD on the 1-dimensional lattice of cubes. We are looking for a continuum limit in this system.

The partition function for each cylinder in the chain is given in Eq. (10). The sum over  $U(N)$  irreps can be calculated explicitly:

$$\begin{aligned}
Z_\varepsilon^{(1)}(e^{i\varphi}, e^{i\psi}) &= \sum_{\ell_1 > \dots > \ell_N} e^{-\frac{\varepsilon}{2N}(\sum \ell_i^2 - \frac{N(N^2-1)}{12})} \frac{\det(e^{i\varphi\ell})}{\Delta(e^{i\varphi})} \frac{\det(e^{-i\psi\ell})}{\Delta(e^{-i\psi})} \\
&= \frac{e^{\frac{\varepsilon(N^2-1)}{24}}}{\Delta(e^{i\varphi})\Delta(e^{-i\psi})} \frac{1}{N!} \sum_{\{\ell \in \mathbb{Z}\}} \sum_{\mathcal{P}_1 \mathcal{P}_2} (-1)^{\mathcal{P}_1 \mathcal{P}_2} e^{-\frac{\varepsilon}{2N} \sum \ell_k^2 + i \sum \ell_k (\varphi_{\mathcal{P}_1 k} - \psi_{\mathcal{P}_2 k})} = \\
&= \frac{e^{\frac{\varepsilon(N^2-1)}{24}}}{\Delta(e^{i\varphi})\Delta(e^{-i\psi})} \sum_{\{h \in \mathbb{Z}\}} \sum_{\mathcal{P}} (-1)^{\mathcal{P}} \left(\frac{N}{\varepsilon}\right)^{\frac{N}{2}} e^{-\frac{N}{2\varepsilon} \sum (\varphi_k + 2\pi h_k - \psi_{\mathcal{P}k})^2} \quad (18)
\end{aligned}$$

where we have used the Poisson resummation formula. In the continuum limit, the areas of the cylinders,  $\varepsilon$ , tend to 0:  $\varepsilon \ll 1/N$ . Thus we find the  $N$  fermion kinetic term in the path integral. We can take into account the winding numbers  $h_k$  by considering the angles  $\varphi_k$  and  $\psi_k$  as unrestricted continuous variables. It is well known that QCD<sub>2</sub> on a cylinder is equivalent to free fermions [17].

For each disk we have the heat-kernel, which we have to expand up to the first order in  $\varepsilon$

$$G_A(e^{i\varphi}) = 1 + \lambda \varepsilon N \sum_{k=1}^N \cos \varphi_k + 0(\varepsilon^2) \quad (19)$$

in order to have a proper continuum limit. In Eq. (19) we have neglected all representations with more than one box in the Young table. It means that we allow only for the 1-fold coverings of the disks. To do it, we have to tend the areas of the disks,  $A$ , to the infinity as:  $e^{-A} \equiv \lambda \varepsilon \ll 1/N$ .

In the continuum limit, we find the fermionic path integral

$$\mathcal{Z} = \int \prod_{k=1}^N \mathcal{D}\varphi_k \exp \int dt N \sum_{k=1}^N \left( -\frac{1}{2} \dot{\varphi}_k^2 + \lambda \cos \varphi_k \right) \quad (20)$$

This problem is equivalent to solving the following Schrödinger equation

$$\frac{\partial^2}{\partial \varphi^2} \psi_n(\varphi) + 2N^2(e_n + \lambda \cos \varphi) \psi_n(\varphi) = 0 \quad (21)$$

with the periodic boundary conditions  $\psi_n(\varphi + 2\pi) = \psi_n(\varphi)$ .

The  $N$ -fermion wave function is given by the Slater determinant

$$\Psi(\varphi_1, \dots, \varphi_N) = \frac{1}{N!} \det[\psi_i(\varphi_j)] \quad (22)$$

with the ground state energy equal to the sum of  $N$  lowest levels:

$$E = N \sum_{n=1}^N e_n \quad (23)$$

The finite temperature free energy is simply

$$F(\mu, \beta) = \sum_{n=1}^{\infty} \log(1 + e^{N(\mu - \beta e_n)}) \quad (24)$$

where  $N\mu$  is a chemical potential and  $\beta$  is an inverse temperature.

The spectrum of Eq. (21) is discrete and the one-particle stationary states are described by the Mathieu wave functions. However, we are interested only in the large  $N$  limit, where the stringy interpretation exists. Therefore, we can simplify considerably the problem by treating Eq. (21) quasiclassically.

The large  $N$  wave functions are

$$\psi_n(\varphi) \approx \frac{e^{iNS(\varphi)}}{\sqrt{S'(\varphi)}} \quad (25)$$

where the phase is given by

$$S = \int d\varphi \sqrt{2(e + \lambda \cos \varphi)} \quad (26)$$

and the classical dynamics is described by the equation

$$t = \int \frac{d\varphi}{\sqrt{2(e + \lambda \cos \varphi)}} \quad (27)$$

which solves in elliptic functions. Let us introduce the new variable  $x = \sin \frac{\varphi}{2}$ . Then Eq. (27) takes the form

$$\sqrt{\frac{e+\lambda}{2}} t = \int \frac{dx}{\sqrt{(1-x^2)(1 - \frac{2\lambda}{e+\lambda}x^2)}} \quad (28)$$

whose solution is the elliptic sinus:  $x = \text{sn}(\sqrt{\frac{e+\lambda}{2}}t)$  with the modulus  $k = \sqrt{\frac{2\lambda}{e+\lambda}}$ .

The quasiclassical quantization gives the equation

$$\operatorname{Re} \int_{-\pi}^{+\pi} d\varphi \sqrt{2(e_n + \lambda) - 4\lambda \sin^2 \frac{\varphi}{2}} = \begin{cases} \pi \frac{n+\frac{1}{2}}{N}, & |e_n| < \lambda \\ 2\pi \frac{n}{N}, & e_n > \lambda \end{cases} \quad (29)$$

The levels  $e_n > \lambda$  are twice degenerate.

Let us rescale the energy  $e \rightarrow \lambda e$  and introduce the density of energy levels

$$\rho(e) = \frac{1}{\sqrt{\lambda}N} \frac{\partial n}{\partial e} = \frac{2}{\pi} \sqrt{\frac{2}{e+1}} \operatorname{Re} K\left(\sqrt{\frac{2}{e+1}}\right) \quad (30)$$

where  $K(k)$  is the complete elliptic integral of the first kind. This expression is valid for all values of  $e$  and  $\lambda$ . If the rescaled energy obeys  $|e| < 1$ , it is convenient to introduce the inverse modulus:  $\tilde{k} = \frac{1}{k} = \sqrt{\frac{e+1}{2}}$ , and then we find

$$\rho(e) = \begin{cases} \frac{2}{\pi} k K(k), & e > 1 \\ \frac{2}{\pi} K(\tilde{k}), & |e| < 1 \end{cases} \quad (31)$$

With the chosen normalization, we find the parametric representation for the ground state energy  $\mathcal{E} = E/N^2$

$$\lambda^{-3/2} \mathcal{E} = \int_{-1}^{e_F} de e \rho(e) \quad \lambda^{-1/2} = \int_{-1}^{e_F} de \rho(e) \quad (32)$$

where  $e_F$  is a Fermi level. We are looking for a singularity of  $\mathcal{E}(\lambda)$ . It is convenient to differentiate  $\lambda^{-3/2} \mathcal{E}$  twice with respect to  $\lambda^{-1/2}$  [14], then

$$\frac{\partial^2 \lambda^{-3/2} \mathcal{E}}{(\partial \lambda^{-1/2})^2} = \frac{1}{\rho(e_F)} \quad (33)$$

The critical point corresponds to  $e_F = 1$ . Using the standard formulas we find the asymptotics

$$\rho(e) = \begin{cases} \frac{1}{\pi} \log \frac{32}{1-e} + \frac{1}{4\pi} (\log \frac{32}{1-e} - 2)(1-e) + O((1-e)^2), & 0 < 1-e \ll 1 \\ \frac{1}{\pi} \log \frac{32}{e-1} + O((e-1)^2), & 0 < e-1 \ll 1 \end{cases} \quad (34)$$

Thus we find the third order phase transition.

## 4 Discussion

The only universal feature of the model considered in the previous section is the  $c = 1$  string phase transition, which takes place when the Fermi level reaches the maximum of the potential [14]. Its existence and the type of the singularity do not depend upon



a choice of the Boltzmann weight (any periodic function has a maximum). Therefore, whatever a lattice action would be, it produces in the continuum limit the non-critical Bose string. We associate this universal stringy behavior with a glueball threshold. If multicolor QCD can indeed be reformulated as a string model, the lowest glueball has to represent a closed-string state.

At a particle threshold, the classical dynamics of a field theory becomes effectively 1-dimensional, simply because the energy is of the order of a mass. Of course, it is not true quantum mechanically. However, in string theory, quantum loop effects are associated with higher topologies. Therefore, we expect that, at the tree level, the one-dimensional string dynamics could correctly describe some universal features of higher-dimensional gauge models.

Another argument in favor of this conclusion is provided by the Fateev-Kazakov-Wiegmann exact solution of Principal Chiral Field at large  $N$  [15]. They have found that the threshold singularity of the PCF free energy is identical to the one in the  $c = 1$  matrix model. To make the parallel between PCF and the model considered in the present paper more transparent, let us consider the infinite 2-dimensional lattice of cubes. One can imagine it as two parallel square lattices whose vertices are connected pairwise by “vertical” links. Let us consider the standard gauge theory on this “two-layer” lattice. To find a continuum limit, we have to introduce different coupling constants for “vertical” and “horizontal” plaquettes,  $g_v$  and  $g_h$  respectively. If  $g_v = \infty$ , the model is equivalent to 2 non-interacting copies of QCD<sub>2</sub>. If  $g_h = 0$ , we find a lattice regularization of PCF. Therefore, in the continuum limit, we find the action

$$\mathcal{A} = \int d^2x \, N \text{tr} \left( \frac{1}{2g_v} (\nabla_\alpha^A \phi) (\bar{\nabla}_\alpha^B \phi^{-1}) + \frac{1}{2g_h} (F_{\mu\nu}^A)^2 + \frac{1}{2g_h} (F_{\mu\nu}^B)^2 \right) \quad (35)$$

where  $\phi(x) \in SU(N)$  is PCF;  $\nabla_\mu^A = \partial_\mu + iA_\mu$  ( $\nabla_\mu^B = \partial_\mu + iB_\mu$ ) are the covariant derivatives and  $F_{\mu\nu}^A = [\nabla_\mu^A, \nabla_\nu^A]$  ( $F_{\mu\nu}^B = [\nabla_\mu^B, \nabla_\nu^B]$ ) are the curvature tensors for 2 copies of gauge field,  $A_\mu$  and  $B_\mu$ , respectively.

The model simplifies in the axial gauge  $A_1 = B_2 = 0$ . After integrating out the gauge fields, one finds the effective action for  $\phi(x)$

$$\mathcal{A}^{eff} = \int d^2x \, N \text{tr} \left\{ \frac{1}{2g_v} |\partial_\alpha \phi|^2 + \frac{g_h}{2g_v^2} \left( J_1^L \frac{1}{\partial_2^2} J_1^L + J_2^R \frac{1}{\partial_1^2} J_2^R \right) \right\} \quad (36)$$

where  $J_\alpha^L = \phi^{-1} \partial_\alpha \phi$  and  $J_\alpha^R = \partial_\alpha \phi \phi^{-1}$  are the left and right invariant currents. This model possesses the main qualitative features of QCD: asymptotic freedom, confinement and non-trivial glueball spectrum. Unfortunately, it looks too complicated to be solved exactly. Fateev, Kazakov and Wiegmann have investigated PCF in the homogeneous external gauge field which fixes an energy scale. It is very plausible that, at the threshold, the model (35) behaves identically to this simplified one. If we accept this hypothesis, it will presumably mean that, despite the sum-over-surfaces formulation of lattice gauge theory looks highly non-trivial [16, 17], what shows in the continuum limit of multicolor QCD is

the simplest non-critical string. However, in higher dimensions, it becomes tachionic thus disappearing and leaving us to guess what really happens in physics.

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